

# Force identification using mixed penalty functions

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**ABSTRACT:** The identification of system parameters using forward approaches is not always practical due to the rising complexity of modern structures, leaving no chance for direct parameter measurements. In contrast to forward methods, inverse techniques have been gaining popularity, since the advent of high performing computers. This approach consists of the computation of input parameters of a system, with known “output data” and the “system model”. When the number of equations (sensors) becomes lower than the amount of unknowns (input parameters), the problem becomes underdetermined and most of the time highly sensitive to input perturbations, leading to infinity of existing solutions. The discrete force identification problem in mechanics consists of estimating a set of two unknown parameters based on measured structural responses: the impact force locations and their corresponding time history. In this article we propose a new approach to identify forces on structures, by making use of an iterative optimization technique. This algorithm creates a new mathematical setting for the inverse problem, and then solves it using a mixed cost function of group-penalized least squares. We also briefly mention some of the most famous existing methods for inverse problems in frequency domain, such as classical pseudo-inverse, Iterative weighted pseudo-inverse, regularization and penalized techniques. This study shows that the location and time history of discrete forces applied on a beam structure can be better estimated using this new iterative and penalization technique (G-FISTA).

**KEYWORDS:** Modal analysis, inverse problem, load identification, mixed  $l_p$ -norm, optimization, sparse solution.

## 1 INTRODUCTION

Force identification using structural vibration data has attracted a lot of interest in the industry. The inverse load identification using response data is especially of interest in civil engineering and structural mechanics. Buildings, wind turbines and stadiums are some of the examples where the dynamics of the structure should be taken into consideration for reliable construction. In the fatigue life assessment of the structure, both material properties and load characteristics are essential parameters. Therefore, the time history of external forces is an important quantity in the forecasting of the remaining lifetime. In many practical applications the measurement of the external loads is either limited or not possible due to sensors limitations and the unknown nature of the external forces; Many attempts were made by engineers to solve this problem by using indirect measurement techniques [1]. As part of the “Inverse Problem” family, this approach consists of the computation of input parameters of a system, with known output data and the system model.

The “inverse” problem is defined in contrast to the well-defined “forward” problem, where the system outputs are computed directly using the inputs and the system model. For most modern and complex structures, the inverse method gives a broad solution possibilities to correctly estimate the applied dynamic loads, because the forward approach is not able to deal with complex structures or load configurations, due to the fact that no measuring points can be available to directly measure the structure’s (reaction) loads. In an inverse approach, some other parameters are measured instead of the

force itself, because of their accessibility to the user, such as acceleration or strain data. (See Figure 1.)

Due to its mathematical characteristics, solving an inverse problem is in general not straightforward and most often leads to a situation with an infinite amount of solutions. Inverse problems are in most cases not “well-posed”. In 1902, *J. Hadamard* formulated the concept of the well-posedness (properness) of problems for differential equations. A problem is called well-posed in the sense of *Hadamard* if there exists a unique solution to this problem that continuously depends on its data. As soon as the number of equations (observations) becomes smaller than the amount of unknowns, the problem becomes underdetermined, and infinity of solutions will exist. For example, in the specific case of mechanical loads, the forces acting on the structure make it vibrate on a (usually) wide frequency band range depending on the force intensity. The behavior of the structure with respect to the tat specific excitation force (e.g. impact) would be reproduced using another force configuration. In other words, a wide range of solutions will exist, since they result the same output data (vibration) on the structure.

Ill-posed problems suffer from two main problems: either the amount of information gathered at system’s output is not enough to reproduce the inputs (less equations than unknowns, underdetermined), or, the system is highly sensitive to the input perturbations which makes the problem ill-conditioned, even if the problem is not (at first sight) ill-posed. It is important to keep in mind that there is no universal method for solving ill-posed problems. In every specific case, the main trouble (instability) has to be tackled in its own way.

Recently, with the advent of powerful computers, inverse and ill-posed problems started to gain popularity very rapidly. In mechanics, the load identification aims to estimate forces on the basis of measured structural response, and the dynamic model of the structure's behavior. The gathered response data needs to be processed in order to compute a good estimation of the real loads applied on the structure. The inverse calculation cannot be solved unless the behavior of the structure is known. For an inverse problem in general, the knowledge of a reliable, accurate and well-predicting "structural model" is crucial. A more robust structural model will yield to a better force identification. Finding a suited structural model of the structure is one of the most important steps in load identification. As all the (inversely) estimated solutions satisfy the requirements of the problem and the structural model (input-output relation), the new challenge will be to select the most realistic one among all the possibilities.

In this article we are going to use the penalized least squares approach to solve our inverse problem. Penalized least squares are an effective method proposed to solve ill-posed systems of linear equations. In this approach one solves the usual LS problem, but by restricting its solution.

Consider a general system of linear equations:  $Ax = b$ , where the matrices  $A$ ,  $x$  and  $b$  represent respectively the model, the inputs and the measured outputs of a system. The classical way to find the inputs  $x$  is the least squares approach, which solves the following problem:

$$\operatorname{argmin}_x \|Ax - b\|_2^2 \quad (1)$$

where the  $l_p$ -norm is defined as:

$$\|\vec{f}\|_2^2 = \left( \sum_{i=1}^n |f_i|^p \right)^{1/p}$$

The least squares method works fine for situations where the problem is either equally-determined or over-determined. But as we have said with ill-posedness this optimization problem cannot be solved. To overcome this problem, one can restrict the solution of previous optimization problem by adding an extra constraint, i.e. to solve the following problem instead:

$$\operatorname{argmin}_x \|Ax - b\|_2^2, g(x) < R \quad (2)$$

In accordance with the duality property, one can show for every  $R$  there exists a  $\lambda$  for which, the solution of the above problem and the following problem is the same:

$$\operatorname{argmin}_x \|Ax - b\|_2^2 + \lambda g(x) \quad (3)$$

This problem is called a penalized least squares problem, and the function  $g$  is the penalty function.

Different choices of penalty functions lead to different solutions. Therefore, the choice of  $g$  should be done in a wise way. Tikhonov [2] has considered  $\|x\|_2^2$  as the penalty while Tibshirani [3] considered  $\|x\|_1$  as the penalty function. The latter penalty function would provide a so-called sparse solution, i.e., many of the components of the solution are zero. Following authors like Turlach et al. [4] and Yuan and Lin [5] one can use a mixture of these penalties for special situations

where there exists a sort of structure among the components of the solution  $x$ . The use of physical properties of the applied loads on the structure leads to the selection of a "mixed" penalty function that compromise for the accuracy of the solution versus its sparsity.

In this article we consider the impact source identification problem and we propose a new technique to solve this kind of problems. We briefly mention some of the most famous existing methods for inverse problems in general, such as classical pseudo-inverse, Iterative weighted pseudo-inverse, regularization and penalized techniques. In section 2 this problem will be reformulated for penalized least squares. Then, an appropriate penalty function will be chosen with respect to the problem structure. An iterative algorithm to solve the derived least squares problem will be modified in this section. Section 4 discusses the implementation of the algorithm and in section 5 some examples will be studied. Some discussions are presented in section 6, and finally, the article will be concluded in section 7.

## 2 PROBLEM STATEMENT

Loads acting on a structure can be of different nature. Most generally, structures are exposed to a combination of locally concentrated, together with distributed forces along the structure. (Ex: Wind loads together with impacts) In the scope of this paper, we focus our main interest on discrete forces, acting at several point locations. A very special case of this kind of load is the "impact force", which is studied in details in this contribution.

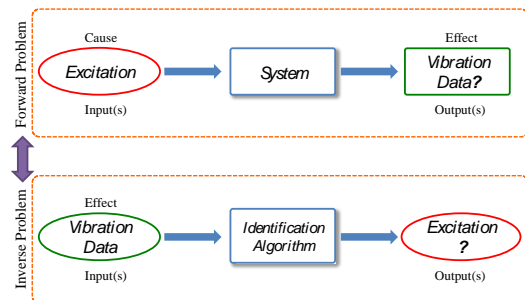


Figure 1. Forward and inverse problem, as defined in frequency domain.

The general force identification problem deals with the reconstruction of unknown dynamic forces acting on a structure from its response in a limited number of sensors and a system model. The (impact) force identification problem consists of estimating a set of two unknown parameters: the impact force locations and their time history. Depending on the problem configuration and the algorithm used, the process of force localization and finding their amplitudes might be simultaneous or not. This choice is directly depending on how the problem is considered and may vary a lot from a study to another. The method used in this paper for load identification has the capability to localize and estimate impact force amplitudes in a single algorithm.

## 2.1 Working domain

In the literature, the load identification problems were treated differently, depending on the choice of working domain. In most cases, “time” domain techniques are used to solve the problem. Recently, there have been great advances in the development of new time domain techniques, especially in the joint input and state identification [7]. The problem is therefore described as a set of linear state equations. These methods make use of an adaptive Kalman filter (on a fusion of measured acceleration and displacement data), together with system’s FEM model to estimate the state of the system, as well as its input parameters. Those methods are fast and can be used in on-line situations. In the other hand, the assumption of the known force locations is a major drawback of those methods. The methods proposed by [8] and [1] use another approaches to localize forces by algorithms based on travel-time calculation or wavelet transform functions.

The use of frequency domain for the calculations in this paper can be justified by several reasons: working with spectrum data is less time-consuming, since the number of frequency parameters is smaller than the number of time domain data (smart frequency selection, see 3.1). Furthermore, by working with the spectra of measured signals, one can easily overview the energy distribution of signals over different frequencies. This approach seems to be efficient in dynamic force identification [9][15].

## 2.2 Hypotheses and model description

The estimation of the forces, which act on a structure during realistic conditions, involves the identification of a model that gives a representative description of the dynamic behavior of the structure during its normal operating conditions. The model should describe the behavior of the structure as accurate as possible.

As discussed in the previous sections, the defined ill-posed problem of impact load identification will be investigated in the frequency domain. In the literature, the use of a *linear* model describing the input-output relations of the mechanical system is of common practice. The structural model can be obtained in different ways: some papers extract the system model using the well-known analytical formulations, such as Euler-Bernoulli or Timoshenko equations, assuming that the studied structure is simplified enough to behave like a pure beam. This approach will be limited to extremely simple structures where the analytical formulation exists [12][14]. In order to eliminate the restrictions of this method, some other papers extract the model using Finite Element simulation, such as in [7]. Like in any other simulation, the numerical models will not be accurate as long as the boundary conditions are not correctly defined in the calculations; because any slight error in the boundary conditions of the domain would result in non-coherent behavior of the structure compared to the real case.

Another approach is to obtain the structural model experimentally, by means of input-output measurements. Although this approach uses the real physical characteristics of the structure into account and solves the previously mentioned limitations, but in the other hand, it cannot be used in an in-operational situation where working loads are already exciting the structure. A lot of effort has been done in order to

deal with in-operational configurations, such as the study of *Parloo* [15] considering Operational Modal Analysis. In our contribution though, the structural model is obtained using Experimental Modal Analysis in lab conditions.

In accordance with authors like [8] and [15], in order to obtain the Frequency Response Function (FRF), the experiment consists of transferring the measured time domain data into the frequency domain, using a fast Fourier transform (FFT).

It is assumed that the system is *time-invariant* and its behavior can be formulated as a *linear* model that takes forces as input and accelerations as output data. In this case, the model is defined as a *receptance* matrix. For each frequency  $f$ , the model is summarized as follows:

$$X(f) = H(f)F(f) \quad , \quad f \in \{f_1, \dots, f_{N_f}\} \quad (4)$$

where  $X(f)$  is the  $n \times 1$  acceleration vector with  $n$  the number of accelerations,  $F(f)$  is the  $k \times 1$  force vector with  $k$  the number of possible applied force locations, and  $H(f)$  is the  $n \times k$  frequency response function (FRF) model matrix.

The  $H(f)$  matrix, which describes the behavior of the system to external excitations, is obtained by the so-called roving hammer test and the  $H1$  method (see [11]). This method assumes the presence of noise only on the output data  $X(f)$  and considers no noise on the applied forces  $F(f)$ . The extended version of the equation (4) is as the following:

$$\begin{pmatrix} H_{11}(f) & H_{12}(f) & \dots & H_{1k}(f) \\ H_{21}(f) & H_{22}(f) & \dots & H_{2k}(f) \\ \vdots & \vdots & \ddots & \vdots \\ H_{n1}(f) & H_{n2}(f) & \dots & H_{nk}(f) \end{pmatrix}_{[n \times k]} \begin{pmatrix} F_1(f) \\ F_2(f) \\ \vdots \\ F_k(f) \end{pmatrix}_{[k \times 1]} = \begin{pmatrix} X_1(f) \\ X_2(f) \\ \vdots \\ X_n(f) \end{pmatrix}_{[n \times 1]} \quad , \quad f \in \{f_1, \dots, f_{N_f}\} \quad (5)$$

where  $H_{ij}(f)$  is defined as the response of the system (acceleration) measured at point  $i$  caused by the system input (force) applied at point  $j$ . The FRF is a 3D  $n \times k \times N_f$  array containing  $H(f)$  in all frequencies, with  $N_f$  the total number of frequencies. The response of a structure to an external load is mostly pronounced at its natural vibration frequencies, called resonance frequency. This information is well preserved in the FRF. Due to harmonic nature of the measured data, the FRF contains complex values. A complex value presents some useful information: the amplitude and the phase angle. These two quantities can completely describe the behavior of the structure over all frequencies. Due to the presence of two variables (amplitude and phase angle), a complex variable is used to represent them together.

## 3 FORCE IDENTIFICATION APPROACH

After computing  $H(f)$  and measuring  $X(f)$ , the next step is to estimate  $F(f)$  in the model of equation (4). There is a system of equations for each value of  $f$ . In a well-posed problem the solution for the frequency  $f$  would be simply:

$$F(f) = (H(f)^T H(f))^{-1} H(f)^T X(f) \quad (6)$$

Although this problem is almost always ill-posed, there are some situations where the direct inversion of the transfer function can produce results, such as in [12]. Because of the instability of the problem, the direct inversion method gave place to the Moore-Penrose pseudo-inverse, introduced and developed by *Penrose* [10]. Despite the uniqueness property of the found solution, the obtained results are not fully satisfying, because the estimated forces are smeared out over all candidate force locations. (Poor localization) This is not an unusual result, because the pseudo-inverse does not produce sparsity in the solution. This method is extremely fast and has always a unique solution, but it lacks the ability to localize discrete forces. Nevertheless, this technique works moderately fine in some especially simple cases.

The force localization process can sometimes be treated separately, as explained in [13], where the force is located by minimizing the solution's entropy. In another paper, [14] the force identification problem is solved by mean of minimization process based on Genetic Algorithms (GA).

Among the non-penalized methods, *Parloo* [15] has suggested a technique called Iterative Weighed Pseudo-Inverse algorithm (IWPI), which is based on an  $l_p$ -norm loss functions. The value of  $p$  tends to zero in an iterative manner until convergence achieved. Convergence is usually achieved for a  $p$  very close to zero; therefore the problem is not convex anymore. Although this technique is relatively fast, we have observed in our experiences that - probably due to non-convexity of the problem with  $p < 1$  - the solution given by this method is sometimes unstable. But in general, it produces satisfying results where the a few number of forces are acting on the structure.

Rather than the methods based on pseudo-inverse and singular value decomposition (SVD), some authors have also considered a penalized model in order to solve this problem. Authors like *Jacquelin et al.* [16] and *Jang et al.* [17] have considered an optimization-based technique, where a least squares loss function is penalized with an  $l_2$ -norm squared penalty (Tikhonov Regularization). In another article, *Romppanen* [18] has considered a total variation penalty function together with the least squares. It seems the least squares loss function gives appropriate results for this problem, but the ill-posedness of the inverse problem highlights the role of an appropriate choice of the penalty function. Therefore, we need to choose a new appropriate penalty function, which suits the nature of this problem. The choice of this penalty function is described in details in the following sections.

### 3.1 New setting for the problem

Considering the system of linear equations in equation (5) and  $N_f$  frequencies, one actually has  $N_f$  systems of linear equations to solve. From the total  $N_f$  frequencies available in the system, only  $N_s$  of them are selected for the first step, which is the localization of forces. Even though the complete system has a limited number of frequency lines, it still has a third dimension that makes the computation more complex.

While the complete range of frequencies is needed to correctly describe the system using FRFs, a "smart" frequency selection can improve the calculation speed, by reducing the number of unknown parameters. In this way, the working

frequency band will be constructed from several segments around the FRF amplitude peaks, where usually most of the information is concentrated. The obtained FRF model will be smaller in size, but still capable of describing the system well. This technique will reduce the number of parameters and reduces the calculation time. It is worth mentioning that the frequency-reduced system is used only for the force localization and as a second step, the force time history will be calculated with the complete amount of frequencies.

In our new setting, we create new 2D matrices ( $\bar{F}$ ,  $\bar{X}$  and  $\bar{H}$ ) based on the existing ones, in order to be able to apply the algorithm of load identification. The  $F$  and  $X$  matrices are created by concatenating respectively the matrices  $F$  and  $X$  of each frequency  $f$ , in form of a column. The new  $\bar{H}$  matrix is obtained by placing all the matrices  $H(f)$  in the diagonal. ( $\bar{H} = \text{diag}(H)$ )

$$\bar{H} = \begin{pmatrix} H(f_1) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & H(f_2) & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & H(f_{N_f}) \end{pmatrix}_{[(N_f \times n) \times (N_f \times k)]} \quad (7)$$

As we have mentioned, using a penalized LS, the choice of the penalty function is very important. In order to make a wise and appropriate choice, consider the following physical facts about the problem we are dealing with:

- Since the forces are applied just at a few discrete points on the beam, the actual forces on most of other locations are assumed to be zero. In this case, one expects the estimated forces also resemble such a pattern. In other words, for each frequency, a *sparse* solution is desirable. Therefore, using an  $l_1$ -penalized model seems to be suitable.
- Because the structure is excited with a hammer, the force will appear in almost every frequency, due to the broadband characteristic of the impact. This means that a zero force will be zero in all frequencies and a non-zero force will be mostly non-zero in all frequencies as well. Therefore, in the frequency domain, such a pattern is expected on the solutions.

$$\bar{F} = \begin{pmatrix} F_1(f_1) \\ \vdots \\ F_n(f_1) \\ F_1(f_2) \\ \vdots \\ F_n(f_2) \\ F_1(f_3) \\ \vdots \\ F_n(f_3) \\ \vdots \\ F_1(f_{N_f}) \\ \vdots \\ F_n(f_{N_f}) \end{pmatrix}_{[(N_f \times n) \times 1]}, \quad \bar{X} = \begin{pmatrix} X_1(f_1) \\ \vdots \\ X_n(f_1) \\ X_1(f_2) \\ \vdots \\ X_n(f_2) \\ X_1(f_3) \\ \vdots \\ X_n(f_3) \\ \vdots \\ X_1(f_{N_f}) \\ \vdots \\ X_n(f_{N_f}) \end{pmatrix}_{[(N_f \times n) \times 1]} \quad (8)$$

Therefore, one can consider a group structure among the components of  $F$ : each candidate force point represents a group, whose components are the forces corresponding to that point along all frequencies. Thus, the number of defined groups is equal to the number of candidate force points, and

he number of members of each group is equal to the number of frequencies. Now one may apply a sparsity-inducing penalty between the groups, and non-sparse penalty within the groups. Considering the size of the problem, we propose to use a  $l_1$  - norm penalty between the groups and a  $l_2$  - norm penalty within the group. Therefore, the appropriate penalty function is the mixed  $l_{1,2}$  - norm penalty function. We will discuss this matter more lately.

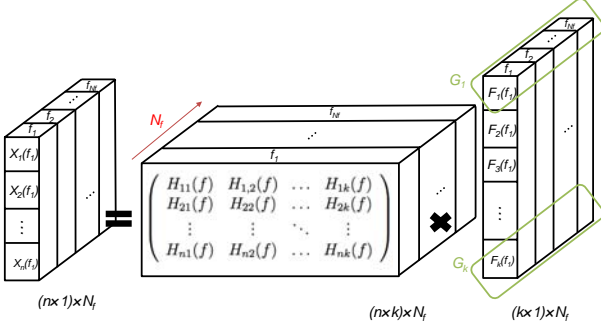


Figure 2. The 3D representation of the input-output model in frequency domain.

Considering the abovementioned settings, the groups are defined as following:

$$G_i = \{F_i(f_1) \dots F_i(f_{N_f})\}, i \in \{1 \dots k\} \quad (9)$$

#### 4 SOLUTION METHOD

After defining  $\bar{X}$ ,  $\bar{F}$ , and  $\bar{H}$  together with groups on  $\bar{F}$  and their corresponding components in  $\bar{F}$  and  $\bar{H}$  the problem that should be solved can be formulated as follows:

$$\hat{F} = \underset{F}{\operatorname{argmin}} \left\{ \frac{1}{2} \|\bar{H}\bar{F} - \bar{X}\|_2^2 + \lambda \|\bar{F}\|_{1,2} \right\} \quad (10)$$

$$\text{with, } \|\bar{F}\|_{1,2} = \sum_{m=1}^n \|F_{G_m}\|_2$$

where  $F_{G_m}$  (with  $m \in \{1 \dots n\}$ ) are the components of  $\bar{F}$  in group  $m$ . In order to solve problem in (10), we modify an efficient iterative algorithm so-called FISTA (Fast Iterative Soft-Threshold Algorithm) in *Beck et al.* [6] for our case. Making these modifications consist of two main aspects:

- The inputs here are complex values while in paper of *Beck et al* [6] they considered to be real.
- The FISTA algorithm needs to be adapted for this group's case. We call this new algorithm groups FISTA, or shortly G-FISTA.

The iterative algorithm G-FISTA consists of the following steps and will converge to  $\hat{F}$  the minimizer of  $\{\|\bar{H}\bar{F} - \bar{X}\|_2^2 + \lambda \|\bar{F}\|_{1,2}\}$ , with a convergence rate of  $O(\frac{1}{k^2})$  (see [6]). The algorithm is represented in the Figure 3.

The projection function  $p_L$  used in the algorithm, is defined as:

$$p_L(u) = \begin{cases} u, & \|u\|_2 < 0 \\ u \frac{L}{\|u\|_2}, & \|u\|_2 \geq 0 \end{cases} \quad (11)$$

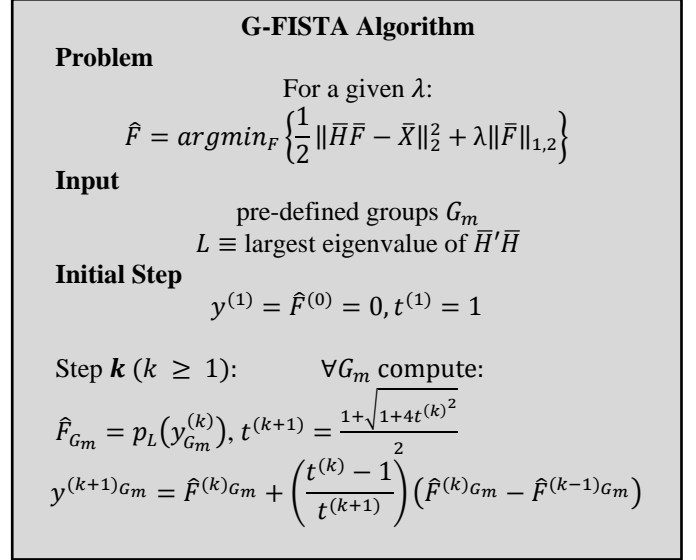


Figure 3. The G-FISTA algorithm steps.

The G-FISTA consists of some matrix multiplication and also computing the largest eigenvalue of a sparse matrix. Implementing of such algorithm in e.g. Matlab is straightforward. The implemented algorithm is available upon a request.

Since displaying the results for all used frequencies is not practical, we use the so-called force index (introduced in [15]) as a representative index of actual and estimated forces, normalized over all groups and frequencies:

$$\beta_i = \frac{\sum_{f=1}^{N_f} \|F_i(f)\|_2^2}{\sum_{j=1}^k \sum_{f=1}^{N_f} \|F_j(f)\|_2^2}, i \in \{1 \dots n\} \quad (12)$$

The advantage of using this parameter is that it creates a non-dimensional representation for the localize forces, and makes the comparison more practical.

As a second step to the force identification process, after localizing the points where forces are present, a reduced system of equations is created, by taking into account only the estimated force locations. Then, this new system is solved using classical methods such as pseudo-inverse for the complete frequency range. As a next step, the time history of the estimated force can be produced by an inverse Fourier transform.

#### 5 SIMULATION EXAMPLE

The following simulation example illustrates the new G-FISTA technique, applied on a cantilever beam. In this section, different load/sensor configurations have been tested in order to validate the algorithm. The simulation consists of two distinct parts:

- Experimental modal analysis to find the beam's FRF. The structure model (full  $H$  matrix) is obtained experimentally, by making use of a roving impact hammer test. A total amount of 7 impact force candidates have been considered ( $k = 7$ ), but only a limited number of acceleration sensors are installed on the beam. ( $n = 1 \dots 3$ ) The experiment has been conducted using a LMS<sup>1</sup> devices and software.
- Localize and estimate the time histories of all possible impact forces acting on the beam, only by using  $k$  number of accelerometers. (under-determined and ill-conditioned inverse problem)

The accelerations matrix  $X$  is computed by applying virtual impact forces  $F$ . In order to reduce the computational cost, we considered the smart frequency selection on the data. For the localization process, only those corresponding to the resonance peaks are taken into account, including a set of extra 20 frequency bins as a margin around each resonance peak. ( $N_s < N_f = 2048$ ) We also transform the problem into a suitable form, as discussed in section 3.1. Thus, the  $n \times N_s$  unknown  $F_i(f)$  are placed in one column vector  $\bar{F}$ , and the  $7 \times N_s$  known  $X_j(f)$  are placed in one column vector  $\bar{X}$ . The matrix  $\bar{H}$  is a block diagonal matrix consisting of all the  $H(f)$  - corresponding to  $F(f)$  and  $X(f)$  - as its diagonal elements. As each point on the beam from 1 to 7 corresponds to a separate group, we obtain:

$$G_1 = \{1, 8, 15, 22, 29, 36, \dots\}$$

$$G_2 = \{2, 9, 16, 23, 30, 37, \dots\}$$

$$G_3 = \{3, 10, 17, 24, 31, 38, \dots\}$$

$$G_4 = \{4, 11, 18, 25, 32, 39, \dots\}$$

$$G_5 = \{5, 12, 19, 26, 33, 40, \dots\}$$

$$G_6 = \{6, 13, 20, 27, 34, 41, \dots\}$$

$$G_7 = \{7, 14, 21, 28, 35, 42, \dots\}$$

When the  $\bar{X}$ ,  $\bar{F}$  and  $\bar{H}$  matrices are created (as mentioned in section 3.1), the mathematical problem will be solved by the G-FISTA algorithm. In order to compare the results of G-FISTA with some other methods we have considered the classical Moore-Penrose pseudo-inverse and the Iterative weighed pseudo-inverse (IWPI) method of *Parloo* [15]. The G-FISTA iterative method makes use of the Bayesian information criterion (BIC) and Mallows's  $C_p$  criteria to select the best value of  $\lambda$ . When the forces are localized on the structure, their time history is calculated by classical pseudo-inverse techniques, considering at this step all the frequencies in the study range. Some results are presented in the following figures. The red boxes on top of the beam represent an impact force and the green ones correspond to the accelerometer locations. The next section is dedicated to the result evaluation and possible discussions.

## 6 DISCUSSION

The comparison of the simulation results presented in figures 4 till 7 lead to the conclusion that among the applied force identification methods, the G-FISTA is more accurate in terms of force localization. The classical pseudo-inverse does not produce satisfying force estimations, since the estimated forces are more distributed than localized. The IWPI is successful most of the time to predict the locations where the impacts are present, but in other hand it is less effective while finding the points without forces. The G-FISTA is particularly strong in localizing forces on the beam, even in multiple impact force configurations (see Figure 6). The effectiveness of G-FISTA's characteristic in localizing forces is more pronounced in the figure 7-(b), where it reproduces almost the same force configuration than the imposed one.

Although its effective force localization capability, the G-FISTA method does not appear to be without weaknesses. It turns out that in situations where there is an impact in all the points of the beam, it fails to correctly estimate force locations. This is mostly due to the fact that this method is especially made for situations where the solution has a sparse characteristic. The cost function that is minimized in this approach privileges solution sets that contain more sparsity patterns. In fact none of G-FISTA and IWPI give an acceptable result in those situations.

While comparing the identification methods, it is also important to take into account the calculation costs. Our simulation shows that the pseudo-inverse method is the fastest but less reliable method among them. The calculation time of G-FISTA method reaches several minutes sometimes depending on the computation power. The IWPI is relatively fast and it might be implemented for on-line force estimations, but its estimated results are not as accurate as the G-FISTA. Penalized cost functions are generally heavier to compute, because the solution is dependent to the lambda parameter, responsible for balancing the solution precision against the sparsity patterns. In this case, the problem is solved for a large amount of lambda values and the best solution is selected based on the BIC or  $C_p$  criteria.

It is also clear that the result quality improves considerably in a direct relation to the number of accelerometers installed on the beam. The simulations also show that G-FISTA method is performing much better comparing to other techniques, especially in situations where a small amount of sensors are installed. (see Figure 7)

## 7 CONCLUSION

In this contribution a novel technique is introduced for force identification by adapting a penalized iterative method called G-FISTA. In order to solve the defined ill-posed problem, first a new mathematical configuration is set, and then the candidate force locations are grouped in the frequency domain. The solution procedure is based on a particular cost function minimization, with an iterative approach. The cost function privileges solutions with a sparsity pattern. Finally the best solution is finally selected by calculating the Bayesian Information Criterion (BIC) and Mallows's  $C_p$  criteria.

Several force/sensor location configurations have been investigated in this research. These simulations prove the efficiency of G-FISTA approach in localizing impact forces

<sup>1</sup> SCADAS System, Leuven Measurement Systems (LMS), Belgium.



on a beam structure, in contrast with other existing methods such as the classic pseudo-inverse or IWPI. In all the simulations, G-FISTA is more accurate and it estimates correct force locations, even in situations where the number of sensors is very low.

Although the calculation cost of G-FISTA algorithm is higher than other force identification techniques compared in this paper, this method has the advantage of producing reliable and accurate force estimations, resulting from the mixed loss function which takes into account the sparse pattern of excitation forces in frequency domain.

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